Class XI Session 2023-24
Subject - Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If D, G and R denote respectively the number of degrees, grades and radians in an angle, then **[1]**

a) $\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$

b) $\frac{D}{90} = \frac{G}{100} = \frac{R}{\pi}$

c) $\frac{D}{100} = \frac{G}{90} = \frac{2R}{\pi}$

d) $\frac{D}{90} = \frac{G}{100} = \frac{R}{2\pi}$

2. If f (x) = $x^3 - \frac{1}{x^3}$, then f (x) + f ($\frac{1}{x}$) is equal to **[1]**

a) $2x^3$

b) $2\frac{1}{x^3}$

c) 1

d) 0

3. If A and B are two events, then P $(\bar{A} \cap B) =$ **[1]**

a) P(B) - P(A ∩ B)

b) $P(\bar{A})P(\bar{B})$

c) 1 - P(A) - P(B)

d) P(A) + P(B) - P(A ∩ B)

4. $\lim_{x\to 1} \frac{\sin \pi x}{x-1}$ is equal to **[1]**

a) $\frac{1}{\pi}$

b) π

c) $-\pi$

d) $-\frac{1}{\pi}$

5. The reflection of the point (4, -13) about the line 5x + y + 6 = 0 is **[1]**

a) (1, 2)

b) (0, 0)

c) (3, 4)

d) (-1, -14)

6. Which of the following is a null set? **[1]**

a) C = ϕ

b) B = {x : x + 3 = 3}

c) D = {0}

d) A = {x : x > 1 and x < 1}

7. If z = $\bar{z}$, then **[1]**

a) none of these

b) z is a complex number

c) z is purely real

d) z is purely imaginary

8. Range of $f(x) = \frac{1}{1-2\cos x}$ is **[1]**

a) $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$

b) $\left[-1, \frac{1}{3}\right]$

c) $\left[\frac{1}{3}, 1\right]$

d) $\left[-\frac{1}{3}, 1\right]$

9. Solve the system of inequalities: $-15 < \frac{3(x-2)}{5} \leq 0$ **[1]**

a) -13 < x < 13

b) -23 < x $\leq$ 2

c) -23 < x < 23

d) -13 < x < 2

10. If $0 < x < \frac{\pi}{2}$, and if $\frac{y+1}{1-y} = \sqrt{\frac{1+\sin x}{1-\sin x}}$, then y is equal to **[1]**

a) $\cot\frac{x}{2} - \tan\frac{x}{2}$

b) $\cot\frac{x}{2}$

c) $\cot\frac{x}{2} + \tan\frac{x}{2}$

d) $\tan\frac{x}{2}$

11. For any set A, (A')' is equal to **[1]**

a) ϕ

b) None of these

c) A

d) A'

12. The two geometric means between the numbers 1 and 64 are **[1]**

a) 4 and 16

b) 8 and 16

c) 2 and 16

d) 1 and 64

13. If A and B are the sums of odd and even terms respectively in the expansion of $(x + a)^n$, then $(x + a)^{2n} - (x - a)^{2n}$ is equal to **[1]**

a) AB

b) 4 AB

c) 4 (A - B)

d) 4 (A + B)

14. If $|x + 2| \leq 9$, then **[1]**

a) x ∈ (- 7 , 11)

b) $x \in (-\infty, -7) \cup (11, \infty)$

c) x ∈ [- 11 , 7]

d) $x \in (-7, -\infty) \cup [\infty, 11)$

15. The set of all prime numbers is **[1]**

a) an infinite set

b) a singleton set

c) none of these

d) a finite set

16. The greatest value of sin x cos x is **[1]**

a) $\frac{1}{\sqrt{2}}$

b) $\frac{\sqrt{3}}{2}$

c) $\frac{1}{2}$

d) 1

17. a + ib $\leq$ c + id is meaningful only when **[1]**

a) none of these

b) $a^2 + d^2 = 0$

c) $b^2 + d^2 = 0$ d) $b^2 + c^2 = 0$

18. Given 11 points, of which 5 lie on one circle, other than these 5, no 4 lie on one circle. Then the number of circles that can be drawn so that each contains at least 3 of the given points is **[1]**

a) 172 b) 156

c) 216 d) None of these

19. **Assertion (A):** The expansion of $(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**

Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The proper measure of dispersion about the mean of a set of observations i.e. standard deviation is expressed as positive square root of the variance. **[1]**

Reason (R): The units of individual observations x_i and the unit of their mean are different that of variance. Since, variance involves sum of squares of $(x - \bar{x})$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the following after removing the modulus **[2]**

$f(x) = |2x - 1|, -1 \leq x \leq 1.$

OR

If $f(x) = \frac{x-1}{x+1}$ then show that: $f(\frac{1}{x}) = -f(x)$

22. Evaluate: $\lim_{x \to 0} \frac{1 - \cos 4x}{x^2}$. **[2]**

23. Find the equation of the parabola whose latus - rectum is 4 units, axis is the line 3x + 4y - 4 = 0 and the tangent at the vertex is the line 4x - 3y + 7 = 0. **[2]**

OR

Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

24. Prove that $(A \cap B')' \cup (B \cap C) = A' \cup B$. **[2]**

25. Using slopes, show that the points (5, 1), (1, -1) and (11, 4) are collinear. **[2]**

Section C

26. Find the domain and range of the real function, $f(x) = \frac{3}{2 - x^2}$. **[3]**

27. Solve the system of inequations: $|X - 1| < 5$, $|x| \geq 2$ **[3]**

28. Show that the points (a, b, c), (b, c, a) and (c, a, b) are the vertices of an equilateral triangle. **[3]**

OR

Verify that (0, 7, -10), (1, 6, -6) and (4, 9, -6) are the vertices of an isosceles triangle.

29. Expand using binomial theorem $\left[1 + \frac{x}{2} - \frac{2}{x}\right]^4, x \neq 0$ **[3]**

OR

Using binomial theorem, prove that $(2^{3n} - 7n - 1)$ is divisible by 49, where $n \in N$

30. Find the real values of x and y, if (1 + i) (x + iy) = 2 - 5z. **[3]**

OR

Express $(1 - 2i)^{-3}$ in the form of (a + ib).

31. Let A and B be two sets. Using properties of sets prove that: **[3]**

i. $A \cap B' = \phi \Rightarrow A \subset B$

ii. $A' \cup B = U \Rightarrow A \subset B$

Section D

32. Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9. **[5]**

33. Differentiate If y = $\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}$ show that $\frac{dy}{dx}$ = (sec x tan x + sec x) **[5]**

OR

Find the differential coefficient of sec x, using first principle.

34. If S be the sum, P be the product and R be the sum of reciprocals of n terms in a G.P, prove that $P^2 = \left(\frac{S}{R}\right)^n$. **[5]**

35. If tan $(\pi \cos\theta)$ = cot $(\pi \sin\theta)$ and then prove that cos $\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$ **[5]**

OR

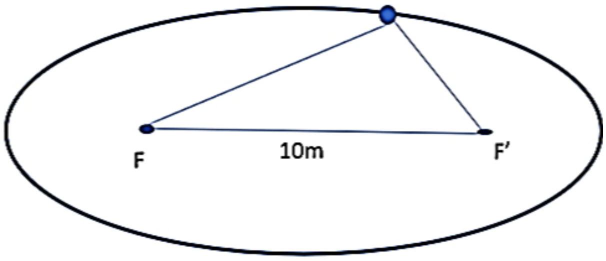
Prove that: cot x + cot$\left(\frac{\pi}{3} + x\right)$ + cot$\left(\frac{2\pi}{3} + x\right)$ = 3 cot 3x.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A farmer wishes to install 2 handpumps in his field for watering.

The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.



(i) Name the curve traced by farmer and hence find the foci of curve.

(ii) Find the equation of curve traced by farmer.

(iii) Find the length of major axis, minor axis and eccentricity of curve along which farmer moves.

OR

Find the length of latus rectum.

37. **Read the text carefully and answer the questions:** **[4]**

You are given some observations as 34, 66, 30, 38, 44, 50, 40, 60, 42, 51.

(i) Find the difference between mean deviation about the mean and mean deviation about the median.

(ii) Calculate the median of the given data.

(iii) The mean deviation about the mean is

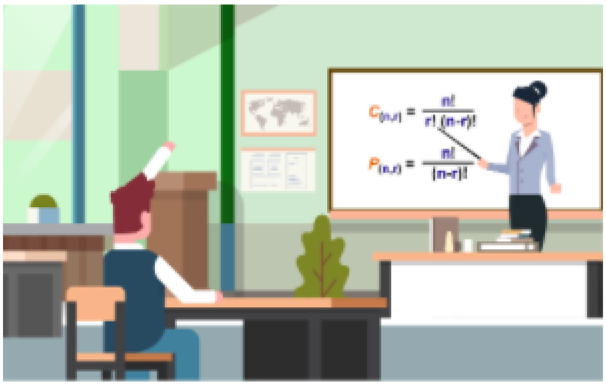
1. 10.0
2. 9.5
3. 9.1
4. 9.0

OR

Calculate the mean of the given data.

38. **Read the text carefully and answer the questions:** **[4]**

During the math class, a teacher clears the concept of permutation and combination to the 11th standard students. After the class was over she asks the students some questions, one of the question was: how many numbers between 99 and 1000 (both excluding) can be formed such that:



(i) How many numbers between 99 and 1000 (both excluding) can be formed such that every digit is either 3 or 7.

(ii) How many numbers between 99 and 1000 (both excluding) can be formed such that without any restriction?

Solution

Section A

1. **(a)** $\frac{D}{90}=\frac{G}{100}=\frac{2R}{\pi}$

Explanation: Let θ be the angle which is measure in degree, radian and grade

We know that 90^{o} = 1 right angle
$\Rightarrow 1^{\circ}=\frac{1}{90}$ right angle
$\Rightarrow \mathrm{D}^{\circ}=\frac{\mathrm{D}}{90}$ right angles
$\Rightarrow \theta=\frac{\mathrm{D}}{90}$ right angle ......(1)
Also we know that ,π radians=2 right angles
$\Rightarrow 1^{c}=\frac{2}{\pi}$ right angle
$\Rightarrow \mathrm{R}=\frac{2}{\pi}\times\mathrm{R}$ right angles
$\Rightarrow \theta=\frac{2}{\pi}\times\mathrm{R}$ right angles ......(2)
Also we know that, 100 grades=1 right angle
$\Rightarrow$ 1 grade= $\frac{1}{100}$ right angle
$\Rightarrow$ G grade= $\frac{G}{100}$ right angles
$\Rightarrow \theta=\frac{G}{100}$ right angles ......(3)
From (1),(2) and (3)
$\frac{\mathrm{D}}{90}=\frac{2\mathrm{R}}{\pi}$
$=\frac{G}{100}$

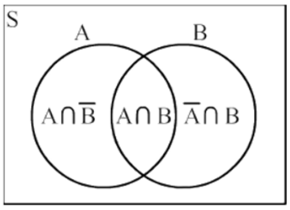
2.

(d) 0

Explanation: Since $f(x)=x^{3}-\frac{1}{x^{3}}$
$f\left(\frac{1}{x}\right)=\frac{1}{x^{3}}-\frac{1}{\frac{1}{x^{3}}}=\frac{1}{x^{3}}-x^{3}$
Hence, $f(x)+f\left(\frac{1}{x}\right)=x^{3}-\frac{1}{x^{3}}+\frac{1}{x^{3}}-x^{3}=0$

3. **(a)** P(B) - P(A ∩ B)

Explanation: $P(B)-P(A\cap B)$



From the diagram, we get A ∩ B and $\overline{A}$ ∩ B are mutually exclusive events such that (A ∩ B) ∪ ($\overline{A}$ ∩ B) = B Therefore by addition theorem of probability we have
$P(A\cap B)+P(\bar{A}\cap B)$ = P(B)
$\therefore P(A\cap B)=P(B)-P(A\cap B)$

4.

(c) $-\pi$

Explanation: $\lim\limits_{x\rightarrow 1}\frac{\sin\pi x}{x-1}$
$=\lim\limits_{h\rightarrow 0}\frac{\sin\pi(1+h)}{(1+h)-1}$
$=\lim\limits_{h\rightarrow 0}\frac{\sin(\pi+\pi h)}{h}$
$=\lim\limits_{h\rightarrow 0}\frac{-\sin\pi h}{h}$
$=\lim\limits_{h\rightarrow 0}-\left(\frac{\sin\pi h}{\pi h}\right)\pi$
$=-\pi$

5.

(d) (-1, -14)

Explanation: Suppose (h, k) be the point of reflection of the given point (4, – 13) about the line 5x + y + 6 = 0.

The mid-point of the line segment joining points (h, k) and (4, – 13) is given by $\frac{h+4}{2}, \frac{k-13}{2}$

This point lies on the given line, thus we have $5\frac{h+4}{2}+\frac{k-13}{2}+6=0$ or 5h + k + 19 = 0 ... (1)

Again the slope of the line joining points (h, k) and (4, –13) is given by $\frac{k+13}{h-4}$.

This line is perpendicular to the given line and therefore, $(-5)\frac{k+3}{h-4}=-1$

This gives 5k + 65 = h – 4 or h – 5k – 69 = 0 ... (2)

On solving (1) and (2), we obtain h = –1 and k = –14.

Therefore thethe point (-1, -14) is the reflection of the given point.

6. **(a)** C = ϕ

Explanation: ϕ is denoted as null set.

7.

(c) z is purely real

Explanation: Let z = x + iy

Now z = $\bar{z}$ $\Rightarrow$ x + iy = x - iy $\Rightarrow$ 2iy = 0 $\Rightarrow$ y = 0

Which means z is purely real.

8. **(a)** $(-\infty,-1]\cup\left[\frac{1}{3},\infty\right)$

Explanation: We know that, $-1\leq\cos x\leq 1$

$\Rightarrow -1\leq-\cos x\leq 1$

$\Rightarrow -2\leq-2\cos x\leq 2$

$\Rightarrow -1\leq 1-2\cos x\leq 3$

Now f(x) $=\frac{1}{1-2\cos x}$ is defined if

$-1\leq 1-2\cos x<0$ or $0<1-2\cos x\leq 3$

$\Rightarrow -1\geq\frac{1}{1-2\cos x}>-\infty$ or $\infty>\frac{1}{1-2\cos x}\geq\frac{1}{3}$

$\Rightarrow \frac{1}{1-2\cos x}\in(-\infty,-1]\cup\left[\frac{1}{3},\infty\right)$

9.

(b) -23 < x $\leq$ 2

Explanation: $-15<\frac{3(x-2)}{5}\leq 0$

$\Rightarrow -15\cdot\frac{5}{3}<\frac{3(x-2)}{5}\cdot\frac{5}{3}\leq 0\cdot\frac{5}{3}$

$\Rightarrow$ -25 < (x - 2) $\leq$ 0 + 2

$\Rightarrow$ -25 + 2 < x - 2 + 2 $\leq$ 2

$\Rightarrow$ -23 < x $\leq$ 2

10.

(d) $\tan\frac{x}{2}$

Explanation: $\frac{y+1}{1-y}=\sqrt{\frac{1+\sin x}{1-\sin x}}$

$\Rightarrow \frac{y+1}{1-y}=\sqrt{\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}$

$\Rightarrow \frac{y+1}{1-y}=\sqrt{\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}$

$\Rightarrow \frac{y+1}{1-y}=\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}$ $[\because 0<x<\frac{\pi}{2}$ $\Rightarrow 0<\frac{x}{2}<\frac{\pi}{4}$, 0 to $\frac{\pi}{4}$ cos x is greater than sin x]

$\Rightarrow \frac{y+1}{1-y}=\frac{\frac{\cos\frac{x}{2}}{\cos\frac{x}{2}}+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{\cos\frac{x}{2}}{\cos\frac{x}{2}}-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$

$\Rightarrow \frac{1+y}{1-y}=\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

Comparing both the sides:
y = tan $\frac{x}{2}$

11.

(c) A

Explanation: We have to find (A')' = ?

Now, A = U\A

⇒ (A')' = (U\A)' = U'\A'

⇒ (A')' = U'\(U\A)

⇒ (A')' = U'\(U\A)

⇒ (A')' = A

12. **(a)** 4 and 16

Explanation: Let the two G.M between 1 and 64 be G_1 and G_2

Therefore, 1, G_1, G_2 and 64 are in G.P.

64 = 1 × r^3

$\Rightarrow r = \sqrt[3]{64}$

⇒ r = 4

⇒ G_1 = ar = 1 × 4 = 4

And, $G_2 = ar^2 = 1 \times 4^2 = 16$

Therefore, 4 and 16 are the required G.M.s

13.

(b) 4 AB

Explanation: If A and B denote respectively the sums of odd terms and even terms in the expansion $(x + a)^n$

Then, $(x + a)^n$ = A + B ... (i)

$(x - a)^n$ = A - B ... (ii)

Squaring and subtraction above equation (ii) from (i) then we get

$(x + a)^{2n} - (x - a)^{2n} = (A + B)^2 - (A - B)^2$

$\Rightarrow (x + a)^{2n} - (x - a)^{2n} = 4AB$

14.

(c) x ∈ [- 11 , 7]

Explanation: $|x + 2| \leq 9$

$\Rightarrow -9 \leq x + 2 \leq 9 \quad [\because |x| \leq a \Leftrightarrow -a \leq x \leq a]$

$\Rightarrow -9 - 2 \leq x + 2 - 2 \leq 9 - 2$

$\Rightarrow -11 \leq x \leq 7$

$x \epsilon [-11, 7]$

15. **(a)** an infinite set

Explanation: Set A = {2, 3, 5, 7,...} so it is infinite.

16.

(c) $\frac{1}{2}$

Explanation: sin x cos x $= \frac{1}{2} \cdot 2\sin x \cos x = \frac{1}{2} \cdot \sin 2x$

But the maximum value of sin 2x is 1.

So the maximum value of sin x cos x = $\frac{1}{2}$

17.

(c) $b^2 + d^2 = 0$

Explanation: We have in the complex numbers system it is meaningless to say one is smaller or bigger than the other since the order relation does not exist in complex numbers.

So a + ib ≤ c + id will be meaningful only when these numbers are real which means their imaginary parts are zero.

⇒ b = 0 and d = 0 ⇒ $b^2 + d^2 = 0$

18.

(b) 156

Explanation: We need at least three points to draw a circle that passes through them.

Now, number of circles formed out of 11 points by taking three points at a time = ${}^{11}C_3$ = 165

Number of circles formed out of 5 points by taking three points at a time = ${}^{5}C_3$ = 10

It is given that 5 points lie on one circle.

∴ Required number of circle = 165 - 10 + 1 = 156

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$

Reason:

$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2 + \ldots + {}^nc_n(1)^{n-n}(-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3} + \ldots (-1)^n n_{c_n}$

Each term will cancel each other

∴ $(1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

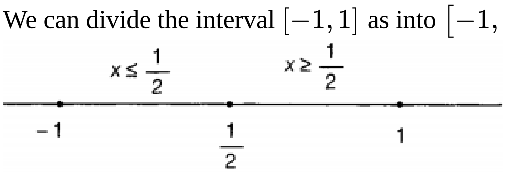
Explanation: Assertion: In the calculation of variance, we find that the units of individual observations x_i and the unit of their mean $\bar{x}$ are different from that of variance, since variance involves the sum of squares of $(x_i - \bar{x})$.

For this reason, the proper measure of dispersion about the mean of a set of observations is expressed as positive square-root of the variance and is called standard deviation.

Section B

21. Putting $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

We can divide the interval $[-1, 1]$ as into $\left[-1, \frac{1}{2}\right]$, $\left[\frac{1}{2}, 1\right]$.



In the interval $\left[-1, \frac{1}{2}\right]$, $2x - 1$ is negative, but the function is positive.

∴ Taking any point x in $\left[-1, \frac{1}{2}\right]$,

$\Rightarrow f(x) = -(2x - 1)$

In the interval $\left[\frac{1}{2}, 1\right]$, $(2x - 1)$ is positive and the function is positive.

∴ Taking any point x in $\left[\frac{1}{2}, 1\right]$,

$\Rightarrow f(x) = 2x - 1$

$$\therefore \quad f(x) = \begin{cases} -(2x-1), & -1 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1 \end{cases}$$

OR

Here we have, f(x) = $\frac{x-1}{x+1}$

Need to prove: $f\left(\frac{1}{x}\right)$ = -f(x)

Now replacing x by $\frac{1}{x}$ we get,

$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1-x}{1+x}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{-(x-1)}{(x+1)}$ = -f(x)

Hence proved.

22. We have: $\lim\limits_{x \to 0}\left[\frac{1-\cos(4x)}{x^2}\right]$

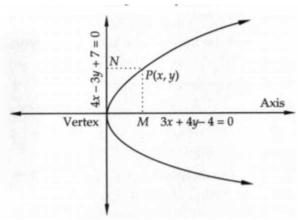
$= \lim\limits_{x \to 0}\left[\frac{2\sin^2 2x}{x^2}\right]$

$= \lim\limits_{x \to 0} 2\left[\frac{\sin 2x}{x} \times \frac{\sin 2x}{x}\right]$

$= \lim_{x \to 0} \left[2 \times \frac{\sin 2x}{2x} \times \frac{\sin 2x}{2x} \times 4\right]$
$= 2 \times 1 \times 1 \times 4$
= 8

23. Let P (x, y) be any point on the parabola.

Construction: Draw PM and PN be perpendiculars from P on the axis and tangent at the vertex respectively.



Then, PM^2 = PN (Latusrectum)

$\Rightarrow \left|\frac{3x+4y-4}{\sqrt{3^2+4^2}}\right|^2 = 4\left|\frac{4x-3y+7}{\sqrt{4^2+(-3)^2}}\right|$

$\Rightarrow$ $(3x + 4y - 4)^2 = 20\ (4x - 3y + 7)$, which is the required equation of the parabola.

OR

Let 2a and 2b be the transverse and conjugate axes and e be the eccentricity. Let the centre be the origin and the transverse and the conjugate axes the coordinate axes. Then, the equation of the hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

We have, 2b = 5 and 2 ae=13

Now, $b^2 = a^2\ (e^2 - 1)$

$\Rightarrow b^2 = a^2 e^2 - a^2$

$\Rightarrow \frac{25}{4} = \frac{169}{4} - a^2 \Rightarrow a^2 = \frac{144}{4} \Rightarrow a = 6.$

Substituting the values of a and b in (i), the equation of the hyperbola is

$\frac{x^2}{36} - \frac{y^2}{25/4} = 1 \Rightarrow 25x^2 - 144y^2 = 900.$

24. To Prove: $(A \cap B')' \cup (B \cap C) = A' \cup B$

LHS = $(A \cap B')' \cup (B \cap C)$

$= (A' \cup (B')') \cup (B \cap C)$ [According to DeMorgan's Law]

$= (A' \cup B) \cup (B \cap C)$

$= ((A' \cup B) \cup B) \cap ((A' \cup B) \cup C)$

$= (A' \cup (B \cup B)) \cap (A' \cup B \cup C)$

$= (A' \cup B) \cap (A' \cup B \cup C)$

$= (A' \cup B)$ = **RHS**

Hence Proved.

25. Suppose A (5, 1), B(1, -1) and C(11, 4) be the given points. Then,

slope of AB = $\frac{(-1-1)}{(1-5)} = \frac{-2}{-4} = \frac{1}{2}$

and slope of BC = $\frac{4-(-1)}{11-1} = \frac{5}{10} = \frac{1}{2}$

$\therefore$ slope of AB = slope of BC

$\Rightarrow$ AB || BC and have a point B in common

$\Rightarrow$ A, B, C are collinear.

Therefore, the given points are collinear.

Section C

26. Given, $f(x) = \frac{3}{2-x^2}$

We know that, f(x) is not defined when $(2 - x^2) = 0$

i.e., $x = \pm\sqrt{2}$

$\therefore$ Domain of $f = R - \{-\sqrt{2}, \sqrt{2}\}$

Also let, $y = \frac{3}{2-x^2} \Rightarrow 2 - x^2 = \frac{3}{y}$

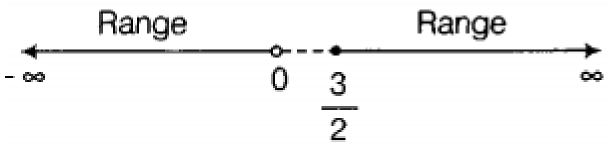
$\Rightarrow \quad x^2 = 2 - \frac{3}{y} \quad \Rightarrow x = \pm\sqrt{2 - \frac{3}{y}} = \pm\sqrt{\frac{2y-3}{y}}$

x is defined, if $\frac{2y-3}{y} \geq 0$ and $y \neq 0,$ i.e.

$(2y-3) \geq 0, y<0$ and $y \neq 0$

$\Rightarrow -\infty < y < 0$ and $\frac{3}{2} \leq y < \infty$

$\therefore$ Range of $f = (-\infty, 0) \cup \left[\frac{3}{2}, \infty\right)$



27. The first inequation is $|x-1| \leq 5$

Using $|x| \leq a \Leftrightarrow -a \leq x \leq a$ we get

$|x-1| \leq 5 \Rightarrow -5 \leq x-1 \leq 5$

$= -4 \leq x \leq 6 \Rightarrow x \in [-4, 6]$ ....(1)

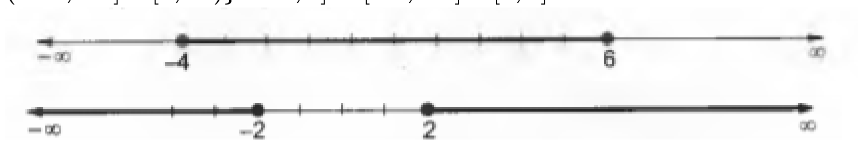
The second inequation is Ixl > 2.

$|x| \geq 2 \Rightarrow x \leq -2$ or $x \geq 2$

$\Rightarrow x \in (-\infty, -2] \cup [2, \infty)$ .....(2)

From (1) and (2) the solution set for the given system is

$(-\infty, -2] \cup [2, \infty)\} \cap -4, 6] = [-4, -2] \cup [2, 6]$



28. Let A (a, b, c), B (b, c, a), and C (c, a, b) be the vertices of △ABC. Then,

AB = $\sqrt{(b-a)^2+(c-b)^2+(a-c)^2}$

$= \sqrt{b^2-2ab+a^2+c^2-2bc+b^2+a^2-2ca+c^2}$

$= \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

$\mathrm{AB} = \sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

BC = $\sqrt{(c-b)^2+(a-c)^2+(b-a)^2}$

$= \sqrt{c^2-2bc+b^2+a^2-2ca+c^2+b^2-2ab+a^2}$

$= \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

$\mathrm{BC} = \sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

CA = $\sqrt{(a-c)^2+(b-a)^2+(c-b)^2}$

$= \sqrt{a^2-2ca+c^2+b^2-2ab+a^2+c^2-2bc+b^2}$

$= \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

$\mathrm{CA} = \sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

$\therefore$ AB = BC = CA

Therefore, △ABC is an equilateral triangle.

OR

Let A(0, 7, -10), B(1, 6, -6) and C(4, 9, -6) be three vertices of triangle ABC. Then

$AB = \sqrt{(1-0)^2+(6-7)^2+(-6+10)^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(4-1)^2+(9-6)^2+(-6+6)^2} = \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$

$AC = \sqrt{(4-0)^2+(9-7)^2+(-6+10)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$

Now AB = BC

Thus, ABC is an isosceles triangle.

29. We have $\left[1+\frac{x}{2}-\frac{2}{x}\right]^4 = \left[1+\left(\frac{x}{2}-\frac{2}{x}\right)\right]^4$

$= {}^4C_0 + {}^4C_1\left(\frac{x}{2}-\frac{2}{x}\right) + {}^4C_2\left(\frac{x}{2}-\frac{2}{x}\right)^2 + {}^4C_3\left(\frac{x}{2}-\frac{2}{x}\right)^3 + {}^4C_4\left(\frac{x}{2}-\frac{2}{x}\right)^4$

$= 1 + 4\left(\frac{x}{2}-\frac{2}{x}\right) + 6\left(\frac{x^2}{4}+\frac{4}{x^2}-2\right) + 4\left(\frac{x^3}{8}-\frac{8}{x^3}-\frac{3x}{2}+\frac{6}{x}\right)$

$+ \left[{}^4C_0\left(\frac{x}{2}\right)^4 - {}^4C_1\left(\frac{x}{2}\right)^3\left(\frac{2}{x}\right) + {}^4C_2\left(\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^2 - {}^4C_3\left(\frac{x}{2}\right)\left(\frac{2}{x}\right)^3 + {}^4C_4\left(\frac{2}{x}\right)^4\right]$

$= 1 + \left(2x-\frac{8}{x}\right) + \left(\frac{3}{2}x^2+\frac{24}{x^2}-12\right) + \left(\frac{x^3}{2}-\frac{32}{x^3}-6x+\frac{24}{x}\right)$

$+ \left(\frac{x^4}{16}-x^2+6-\frac{16}{x^2}+\frac{16}{x^4}\right)$

$= -5-4x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}+\frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}$

OR

To prove: $(2^{3n} - 7n - 1)$ is divisible by 49, where $n \in N$

$(2^{3n} - 7n - 1) = (2^3)n - 7n - 1$

$= 8^n - 7n - 1$

$= (1 + 7)^n - 7n - 1$

Now using binomial theorem..

$\Rightarrow {}^nC_0 1^n + {}^nC_1 1^{n-1} 7 + {}^nC_2 1^{n-2} 7^2 + \ldots\ldots + {}^nC_{n-1} 7^{n-1} + {}^nC_n 7^n - 7n - 1$

$= {}^nC_0 + {}^nC_1 7 + {}^nC_2 7^2 + \ldots\ldots + {}^nC_{n-1} 7^{n-1} + {}^nC_n 7^n - 7n - 1$

$= 1 + 7n + 7^2[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1} 7^{n-3} + {}^nC_n 7^{n-2}] - 7n - 1$

$= 7^2[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1} 7^{n-3} + {}^nC_n 7^{n-2}]$

$= 49[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1} 7^{n-3} + {}^nC_n 7^{n-2}]$

$= 49K$, where $K = ({}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1} 7^{n-3} + {}^nC_n 7^{n-2})$

Now, $(2^{3n} - 7n - 1) = 49K$

Therefore $(2^{3n} - 7n - 1)$ is divisible by 49.

30. $(1 + i)(x + iy) = 2 - 5i$

$\Rightarrow 1(x + iy) + i(x + iy) = 2 - 5i$

$\Rightarrow x + iy + ix + i^2y = 2 - 5i$

$\Rightarrow x + iy + ix - y = 2 - 5i$

$\Rightarrow x - y + i(x + y) = 2 - 5i$

Equating real and imaginary parts we get

$x - y = 2$ ... (i)

$x + y = -5$ ... (ii)

Adding (i) and (ii) we get

$2x = 2 - 5$

$\Rightarrow 2x = -3$

$\Rightarrow x = \frac{-3}{2}$

Substituting the value of x in (i), we get

$\frac{-3}{2} - y = 2$

$\Rightarrow \frac{-3}{2} - 2 = y$

$\Rightarrow y = \frac{-3-4}{2}$

$\Rightarrow y = \frac{-7}{2}$

Hence

$x = \frac{-3}{2}, y = \frac{-7}{2}$

OR

Let $z = (1 - 2i)^{-3}$

$= \frac{1}{(1-2i)^3} = \frac{1}{1-8i^3-6i+12i^2}$ [$\because (a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$]

$= \frac{1}{1-8i^2 \cdot i-6i+12(-1)}$

$= \frac{1}{1+8i-6i-12}$ [$\because i^2 = -1$]

$= \frac{1}{-11+2i} = \frac{1}{-11+2i} \times \frac{-11-2i}{-11-2i}$ [multiplying numerator and denominator by $-11 - 2i$]

$= \frac{-11-2i}{(-11)^2-(2i)^2} = \frac{-11-2i}{121+4}$ [$\because (a - b)(a + b) = a^2 - b^2$]

$= \frac{-11-2i}{125} = \frac{-11}{125} - \frac{2i}{125} = a + ib$ [say]

where, $a = \frac{-11}{125}$ and $b = \frac{-2}{125}$

31. i. We have

$A = (A \cap U)$

$\Rightarrow A = A \cap (B \cup B')$ [$\because B \cup B' = U$]

$\Rightarrow A = (A \cap B) \cup (A \cap B')$ [$\because \cap$ is distributive over union], therefore we get

$\Rightarrow A = (A \cap B) \cup \phi$ [$\because A \cap B' = \phi$]

⇒ A = A ∩ B

∴ A ⊂ B

ii. From (i), we have

A ∩ B' = ϕ

⇔ (A ∩ B')' = ϕ'

⇔ A' ∪ (B')' = U [∵ ϕ' = U]

⇔ A' ∪ B = U [∵ (B')' = B]

Therefore ,A ∩ B' = ϕ ⇔ A' ∪ B = U and, A ∩ B' = ϕ ⇒ A ⊂ B

∴ A' ∪ B = U ⇒ A ⊂ B

Section D

32. We have integers 1, 2, 3,… 1000

∴ Total number of outcomes, n(S) = 1000

Number of integers which are multiple of 2 are 2, 4, 6, 8, 10,… 1000 and it is forming an AP with the common difference of 2

Let p be the number of multiples of 2

Using formula of n^{th} term of AP,

$a_p = a + (p - 1)d$

Here, a = 2, d = 2 and $a_p = 1000$

Putting the value, we get

2 + (p - 1)2 = 1000

⇒ 2 + 2p – 2 = 1000

$\Rightarrow p = \frac{1000}{2}$

⇒ p = 500

Total number of integers which are multiple of 2 = 500

Let the number of integers which are multiple of 9 be n.

Number which are multiples of 9 are 9, 18, 27,…999 and this series is also an AP with common difference of 9.

∴ n^{th} term = 999

Using formula of n^{th} term of AP,

$a_n = a + (n - 1)d$

Here, a = 9, d = 9 and $a_n = 999$

Putting the value, we get

9 + (n – 1)9 = 999

⇒ 9 + 9n – 9 = 999

$\Rightarrow n = \frac{999}{9}$

⇒ n = 111

So, the number of multiples of 9 from 1 to 1000 is 111.

The common multiple of 2 and 9 both are 18, 36,… 990 and these are forming AP too.

Let m be the number of terms in above series.

∴ m^{th} term = 990

Using formula of n^{th} term of AP,

$a_m = a + (m - 1)d$

Here, a = 9 and d = 9

Putting the value, we get

18 + (m – 1)18 = 990

⇒ 18 + 18m - 18 = 990

$\Rightarrow m = \frac{990}{18}$

⇒ m = 55

Number of multiples of 2 or 9

= No. of multiples of 2 + no. of multiples of 9

– No. of multiples of 2 and 9 both

= 500 + 111 - 55

= 556 = n(E)

Required probability $= \frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)} = \frac{556}{1000}$ = 0.556

33. We have to show that $\frac{dy}{dx} = (\sec x \tan x + \sec x)$

where, it is given that

y = $\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}$

$y = \sqrt{\frac{\frac{1}{\cos x}\frac{\sin x}{1}}{\cos x} + \frac{\sin x}{\cos x}} = \sqrt{\frac{1-\sin x}{1+\sin x}}$

u = 1 - sin x, v = 1 + sin x, $x = \frac{1-\sin x}{1+\sin x}$

if z = $\frac{u}{v}$

$\frac{dz}{dx} = \frac{v\times\frac{du}{dx} - u\times\frac{dv}{dx}}{v^2}$

$= \frac{(1+\sin x)\times(-\cos x)-(1-\sin x)\times(\cos x)}{(1+\sin x)^2}$

$= \frac{-\cos x-\sin x\cos x-\cos x+\sin x\cos x}{(1+\sin x)^2}$

$= \frac{-2\cos x}{(1+\sin x)^2}$

According to the chain rule of differentiation

$\frac{dy}{dx} = \frac{dy}{dz}\times\frac{dz}{dx}$

$= \left[-\frac{\cos x}{1}\times\left(\frac{1-\sin x}{1}\right)^{-\frac{1}{2}}\right]\times\left[\frac{1}{(1+\sin x)^{2-\frac{1}{2}}}\right]$

$= \left[\cos x\times(1+\sin x)^{-\frac{1}{2}}\right]\times(1-\sin x)^{-\frac{3}{2}}\times\left(\frac{1+\sin x}{1+\sin x}\right)^{\frac{3}{2}}$

Multiplying and dividing by $(1+\sin x)^{\frac{3}{2}}$

$= \left[\cos x\times(1+\sin x)^{\frac{2}{2}-\frac{1}{2}}\right]\times(1-\sin x)^{-\frac{2}{2}}\times\left(\frac{1}{1+\sin x}\right)^{\frac{3}{2}}$

$= \left[\cos x\times(1+\sin x)^{\frac{2}{2}-\frac{1}{2}}\right]\times(1-\sin x)^{-\frac{2}{2}}\times(1+\sin x)^{-\frac{2}{2}}$

$= \left[\cos x\times(1+\sin x)^{1}\right]\times\left(1-\sin^2 x\right)^{-\frac{3}{2}}$

$= \left[\cos x\times(1+\sin x)^{1}\right]\times\left(\cos^2 x\right)^{-\frac{3}{2}}$

$= \left[\cos x\times(1+\sin x)^{1}\right]\times(\cos x)^{-3}$

$= \left[(1+\sin x)^{1}\right]\times(\cos x)^{-3+1}$

$= \frac{1+\sin x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}\times\frac{1+\sin x}{\cos^2 x}$

$= \sec x\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)$

= sec x (sec x + tan x)

Hence proved

OR

We have, f(x) = sec x

By using first principle of derivative,

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0}\frac{\sec(x+h)-\sec x}{h}$

$= \lim_{h\to 0}\frac{\frac{1}{\cos(x+h)}-\frac{1}{\cos x}}{h}$

$= \lim_{h\to 0}\frac{\cos x-\cos(x+h)}{h\times\cos x\cdot\cos(x+h)}$

$= \lim_{h\to 0}\left[\frac{-2\sin\left(\frac{x+x+h}{2}\right)\cdot\sin\frac{(x-x-h)}{2}}{h\cdot\cos x\cdot\cos(x+h)}\right]$

$\left[\because \cos C-\cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$= \lim_{h\to 0}\left[\frac{-2\sin\left(x+\frac{h}{2}\right)\cdot\left(-\sin\frac{h}{2}\right)}{h\cdot\cos x\,\cos(x+h)}\right]$

$= \lim\limits_{h\to 0} \frac{\sin\left(x+\frac{h}{2}\right)}{\cos(x+h)\cdot\cos x} \cdot \lim\limits_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}$

$= \frac{\sin x}{\cos^2 x} \times (1) = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

= tanx×secx

34. Let G. P. be a, ar, ar^2, ...

Where r < 1

$S = \frac{a(1-r^n)}{1-r}$

$R = \frac{1}{a} + \frac{1}{ar} + - - + n$

$= \frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^n - 1\right]}{\frac{1}{r}-1}$ $[\because r < 1 \text{ then } \frac{1}{r} > 1$

$= \frac{1}{a} \cdot \frac{1-r^n}{r^n} \cdot \frac{r}{1-r}$

$= \frac{1-r^n}{ar^{n-1}(1-r)}$

P = a.ar.ar^2 ... ar^{n-1}

= an.$r^{1+2+----+(n-1)}$

$= a^n . r^{\frac{n(n-1)}{2}}$

$= a^n . r^{\frac{n(n-1)}{2}}$

L.H.S. = P^2R^n

$= a^{2n} . r^{n(n-1)} \cdot \frac{(1-r^n)^n}{ar^{n-1}(1-r)^n}$

= S^n

$P^2 = \left(\frac{S}{R}\right)^n$

Hence proved.

35. Given,

$tan\ (\pi \cos\theta) = cot\ (\pi \sin\theta)$

$\Rightarrow \frac{\sin(\pi\cos\theta)}{\cos(\pi\cos\theta)} = \frac{\cos(\pi\sin\theta)}{\sin(\pi\sin\theta)}$

$\Rightarrow$ sin $(\pi\cos\theta)$ × sin $(\pi\sin\theta)$ = cos $(\pi\sin\theta)$ × cos $(\pi\cos\theta)$

$\Rightarrow$ cos $(\pi\cos\theta)$ ×cos $(\pi\sin\theta)$ - sin $(\pi\cos\theta)$ × sin $(\pi\sin\theta)$ = 0

$\Rightarrow$ cos $[\pi\cos\theta + \pi\sin\theta]$ = 0

$[\because cos\, x \times cos\, y - sin\, x \times sin\, y = cos(x+y)\,]$

$\Rightarrow$ cos $(\pi\cos\theta + \pi\sin\theta)$ = cos $\left(\pm\frac{\pi}{2}\right)$ [∵ cos $\left(\pm\frac{\pi}{2}\right)$ = 0]

$\Rightarrow \pi\cos\theta + \pi\sin\theta = \pm\frac{\pi}{2} \Rightarrow$ cosθ + sinθ = $\pm\frac{1}{2}$

On multiplying both sides by $\frac{1}{\sqrt{2}}$, we get

$\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \pm\frac{1}{2\sqrt{2}}$

$\Rightarrow$ cosθ × cos $\frac{\pi}{4}$ + sinθ × $\frac{\pi}{4} = \pm\frac{1}{2\sqrt{2}}$

cos $\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

$[\because cos\, x \times cos\, y + sin\, x \times sin\, y = cos(x-y)\,]$

Hence proved.

OR

We have to prove cot x + cot$\left(\frac{\pi}{3}+x\right)$ + cot$\left(\frac{2\pi}{3}+x\right)$ = 3 cot 3x.

LHS = cot x + cot$\left(\frac{\pi}{3}+x\right)$ + cot$\left(\frac{2\pi}{3}+x\right)$

We know,

cot$\left(\frac{2\pi}{3}+x\right)$ = cot$\left(\pi - \left(\frac{\pi}{3}-x\right)\right)$ = -cot$\left(\frac{\pi}{3}-x\right)$ ... (as - cotθ = cot (180^o - θ)

Hence the above LHS becomes

= cot x + cot$\left(\frac{\pi}{3}+x\right)$ - cot$\left(\frac{\pi}{3}-x\right)$

$= \frac{1}{\tan x} + \frac{1}{\tan\left(\frac{\pi}{3}+x\right)} - \frac{1}{\tan\left(\frac{\pi}{3}-x\right)}$

$= \frac{1}{\tan x} + \left(\frac{1-\tan x\tan\frac{\pi}{3}}{\tan\frac{\pi}{3}+\tan x}\right) - \left(\frac{1+\tan x\tan\frac{\pi}{3}}{\tan\frac{\pi}{3}-\tan x}\right)$ ... [∵ tan(A + B) = $\left(\frac{\tan A+\tan B}{1-\tan A\tan B}\right)$ and tan(A - B) = $\left(\frac{\tan A-\tan B}{1+\tan A\tan B}\right)$]

$= \frac{1}{\tan x} + \left(\frac{1-\sqrt{3}\tan x}{\sqrt{3}+\tan x}\right) - \left(\frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x}\right)$

$= \frac{1}{\tan x} + \left(\frac{(1-\sqrt{3}\tan x)(\sqrt{3}-\tan x)-(1+\sqrt{3}\tan x)(\sqrt{3}+\tan x)}{(\sqrt{3}+\tan x)(\sqrt{3}-\tan x)}\right)$

$= \frac{1}{\tan x} + \left(\frac{(\sqrt{3}-\tan x-3\tan x+\sqrt{3}\tan^2 x)-(\sqrt{3}+3\tan x+\tan x+\sqrt{3}\tan^2 x)}{(3-\tan^2 x)}\right)$

$= \frac{1}{\tan x} + \left(\frac{(0-4\tan x-4\tan x+0)}{(3-\tan^2 x)}\right)$

$= \frac{1}{\tan x} - \left(\frac{8\tan x}{((3-\tan^2 x))}\right)$

$= \left(\frac{(3-\tan^2 x)-8\tan^2 x}{\tan x(3-\tan^2 x)}\right) = \left(\frac{(3-\tan^2 x)-8\tan^2 x}{\tan x(3-\tan^2 x)}\right)$

$= 3\left(\frac{1-3\tan^2 x}{(3\tan x-\tan^3 x)}\right)$

$= 3 \times \frac{1}{\tan 3x}$ ... (as $\tan 3x = \frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$)

= cot 3x

LHS = RHS

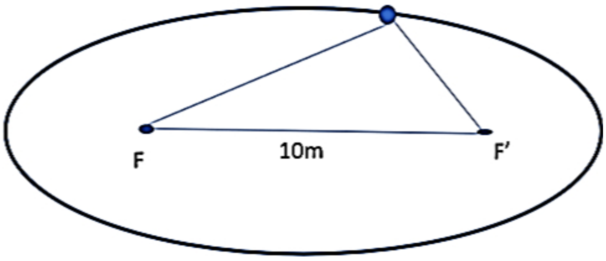
Hence proved.

Section E

36. **Read the text carefully and answer the questions:**

A farmer wishes to install 2 handpumps in his field for watering.

The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.



(i) The curve traced by farmer is ellipse. Because An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Two positions of hand pumps are foci Distance between two foci = 2c = 10 Hence c = 5 Here foci lie on x axis & coordinates of foci = $(\pm c, 0)$

Hence coordinates of foci =$(\pm 5, 0)$

(ii) $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Sum of distances from the foci = 2a

Sum of distances between the farmer and each hand pump is = 26 = 2a

$\Rightarrow$ 2a = 26 $\Rightarrow$ a = 13 m

Distance between the handpump = 10m = 2c

$\Rightarrow$ c = 5 m

$c^2 = a^2 - b^2$

$\Rightarrow 25 = 169 - b^2$

$\Rightarrow b^2 = 144$

Equation is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

(iii) Equation of ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$ comparing with standard equation of ellipse a=13, b= 12 and c= 5 (given)

Length of major axis = 2a = 2 × 13 = 26

Length of minor axis = 2b = 2 $\times$ 12 = 24

eccentricity $e = \frac{c}{a} = \frac{5}{13}$

OR

Equation of the ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$ hence a = 13 and b = 12

length of latus rectum of ellipse is given by $\frac{2\,b^2}{a} = \frac{2 \times 144}{13}$

37. **Read the text carefully and answer the questions:**

You are given some observations as 34, 66, 30, 38, 44, 50, 40, 60, 42, 51.

(i) = mean deviation about the mean - mean deviation about the median

= 9.0 - 8.7

= 0.3

(ii) Number of observation are given calculate mean deviation Mean deviation $\sum \frac{di}{n}$

Here Observation 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

Since Median is the middle number of all the observation

Arrange the numbers in Ascending orders we get 30, 34, 38, 40, 48, 44, 50, 51, 60, 66

Here the number of observation are Even then the middle terms are 42, 44...

Therefore, the median = $\frac{42+44}{2}$

= $\frac{86}{2}$ = 43

(iii)Mean deviation is

x_i	$\lvert d_i \rvert = \lvert x_i - 43 \rvert$
30	13
34	9
38	5
40	3
42	1
44	1
50	7
51	8
60	17
66	23
Total	87

MD = $\frac{1}{n}\sum_{i=1}^{n}|d_i|$

= $\frac{90}{10}$

= 9

OR

The observation are 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

Mean = $\frac{\sum x_i}{n}$

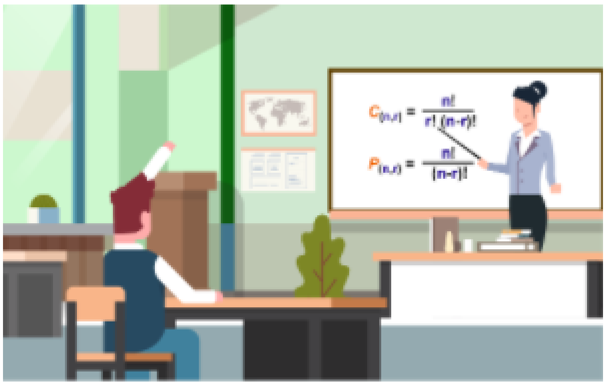
= $\frac{34+66+30+38+44+50+40+60+42+51}{10}$

= $\frac{455}{10}$

= 45.5

38. **Read the text carefully and answer the questions:**

During the math class, a teacher clears the concept of permutation and combination to the 11th standard students. After the class was over she asks the students some questions, one of the question was: how many numbers between 99 and 1000 (both excluding) can be formed such that:



(i) Here we need to get a 3-digit number

Three vacant paces are fixed with 3 or 7. Therefore, by the multiplication principle, the required number of three-digit numbers with every digit 3 or 7 3 or 7 is $2 \times 2 \times 2 = 8$

(ii) Three vacant paces are fixed with all 10 digits, but first place is fixed with 9 digits excluding 0.

Therefore, by the multiplication principle, the required number of three digits numbers without any restriction = $9 \times 10 \times 10 = 900$